mathSUX²

Geometry Cheat Sheet

Notation:

- ≅ congruent
- ~ similar
- Δ triangle
- ∢ angle
- ∥ parallel
- ⊥ perpendicular
- $\overline{AB}$ line segment AB
- $\widehat{AB}$ arc AB

Equation of a Line:

$y = mx + b$

$m = slope = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$

$b = y - intercept$

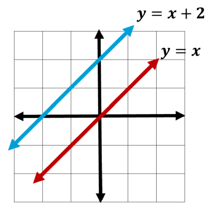
Point Slope Form:

$y - y_1 = m(x - x_1)$

Parallel and Perpendicular Lines:

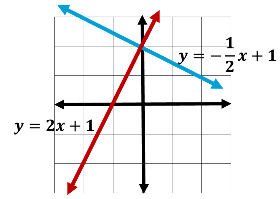
Parallel: Same Slope

$m = m$



Perpendicular: Take negative reciprocal

$m \rightarrow -{}^{1}/_{m}$



Distance Formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula:

$M = (\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2})$

Law of Sins:

$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}$

Law of Cosines:

$c^2 = a^2 + b^2 - 2abcosC$

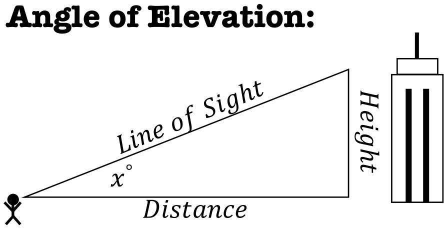
Converting Degrees to Radians:

$ex: 60^\circ \times \frac{\pi}{180} = \frac{\pi}{3}$

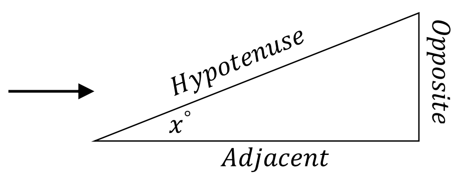
Converting Radians to Degrees:

$ex: \frac{\pi}{3} \times \frac{180}{\pi} = 60^\circ$

Angle of Elevation:



SOH CAH TOA:



$sin(x) = \frac{opposite}{hypotenuse}$

$cos(x) = \frac{adjacent}{hyptenuse}$

$tan(x) = \frac{opposite}{adjacent}$

Inverse Trig. Functions:

$sec(x) = \frac{1}{cos(x)}$

$csc(x) = \frac{1}{sin(x)}$

$cot(x) = \frac{1}{tan(x)}$

Complimentary Angles:

$sin(90^\circ - \theta) = cos(\theta)$

$cos(90^\circ - \theta) = sin(\theta)$

$tan(90^\circ - \theta) = cot(\theta)$

$csc(90^\circ - \theta) = sec(\theta)$

$sec(90^\circ - \theta) = csc(\theta)$

$cot(90^\circ - \theta) = tan(\theta)$

Probability:

n=Total number of objects
r=Number of chosen objects

Permutation: ${}_nP_r = \frac{n!}{(n-r)!}$
(Order matters)

Combinations: ${}_nC_r = \frac{n!}{r!(n-r)!}$
(Order doesn't matter)

Conditional: $P(B|A) = \frac{P(A \cap B)}{P(A)}$

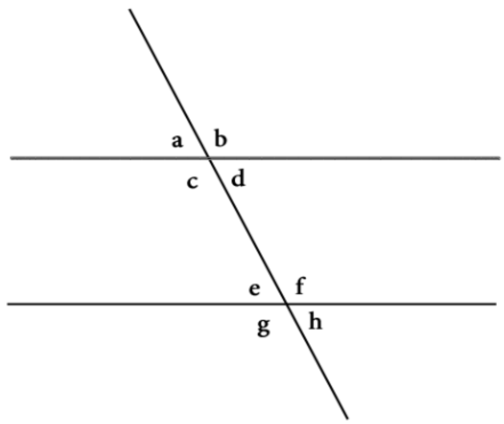
And:
$P(A \cap B) = P(A) \times P(B)$ *(Independent)*

$P(A \cap B) = P(A) \times (B|A)$ *(Dependent)*

Or:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ *(Not Mutually Exclusive)*

$P(A \cup B) = P(A) + P(B)$ *(Mutually Exclusive)*

Transversals: Given two lines are parallel and are cut by a transversal line.



Alternate Interior Angles:
$\measuredangle c = \measuredangle f$ and $\measuredangle d = \measuredangle e$

Alternate Exterior Angles:
$\measuredangle a = \measuredangle h$ and $\measuredangle b = \measuredangle g$

Corresponding Angles:
$\measuredangle a = \measuredangle e, \measuredangle b = \measuredangle f, \measuredangle c = \measuredangle g$, and $\measuredangle d = \measuredangle h$

Supplementary Angles:
$\measuredangle c + \measuredangle e = 180^\circ, \measuredangle d + \measuredangle f = 180^\circ, \measuredangle a + \measuredangle b = 180^\circ,$
$\measuredangle c + \measuredangle d = 180^\circ, \measuredangle e + \measuredangle f = 180^\circ, \measuredangle g + \measuredangle h = 180^\circ$

Properties of a Parallelogram:
1) Opposite sides are parallel.
2) Pairs of opposite sides are congruent.
3) Pairs of opposite angles are congruent.
4) Diagonals bisect each other.
5) Diagonals separate parallelogram into 2 congruent triangles.
6) Interior angles add up to 360°.

The following shapes are all Parallelograms:
1) Square *(also a rhombus and a rectangle)*
2) Rhombus
3) Rectangle

Transformations:
Reflection in the x-axis: $A(x, y) \rightarrow A'(x, -y)$
Reflection in the y-axis: $A(x, y) \rightarrow A'(-x, y)$
Reflection over the line y=x: $A(x, y) \rightarrow A'(y, x)$
Reflection through the origin: $A(x, y) \rightarrow A'(-x, -y)$
Rotation of 90°: $A(x, y) \rightarrow A'(-y, x)$
Rotation of 180°: $A(x, y) \rightarrow A'(-x, -y)$
Rotation of 270°: $A(x, y) \rightarrow A'(y, -x)$
Dilation of *n*: $A(x, y) \rightarrow A'(xn, yn)$
Transformation to the left *m* units and up *n* units: $A(x, y) \rightarrow A'(x - m, y + n)$

Congruent Triangles ≅:

SAS
SSS
AAS
HL –(only for right triangles)
ASA

When proven use: Corresponding parts of congruent triangles are congruent (CPCTC)

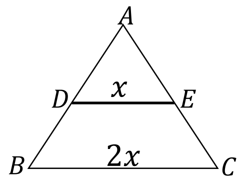
Similar Triangles ~:

AA
SSS
SAS

When proven use: Corresponding sides of similar triangles are in proportion.

Midpoint Triangles Theorem:

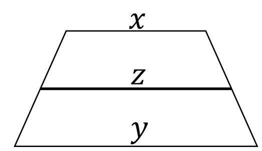
ΔABC has midpoints at point D and point E. When points D and E are connected, the length of $\overline{DE}$ is half the length of base $\overline{BC}$.



Medians of a Trapezoid:

In a trapezoid, the length of median *z* is equal to half the length of the sum of both bases x and y.

$$z = \frac{1}{2}(x + y)$$



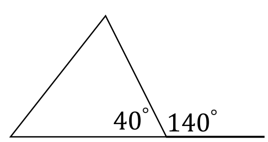
Types of Triangles:

Scalene: No sides are equal.
Equilateral: All sides are equal.
Isosceles: Two sides are equal.

Acute: All angles are < 90°.
Obtuse: There is an angle > 90°.
Right: There is an angle = 90°.

External Angle Triangles Theorem:

When any side of a triangle is extended the value of its angle is supplementary to the angle next to it (adding to 180°). ex:



$$40^\circ + 140^\circ = 180^\circ$$

Volume:

Sphere: $V = \frac{4}{3}\pi r^3$
Cylinder: $V = \pi r^2 h$
Pyramid: $V = \frac{1}{3}bh$
Cone: $V = \frac{1}{3}\pi r^3$
Prism: $V = bh$

Area:

Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$
Triangle: $A = \frac{1}{2}bh$
Rectangle: $A = bh$
Square: $A = s^2$
Circle: $A = \pi r^2$

Perimeter:

Rectangle: $P = 2l + 2w$
Square: $P = 4s$
Circle: $\text{Circumference} = \pi d$

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

Polygon Angle Formulas:

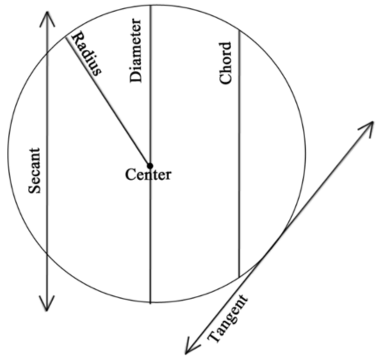
n=number of sides

Value of each Interior Angle: $\frac{180(n-2)}{n}$
Sum of Interior Angles: $180(n-2)$
Value of each Exterior Angle: $\frac{360}{n}$
Sum of Exterior Angles: 360°

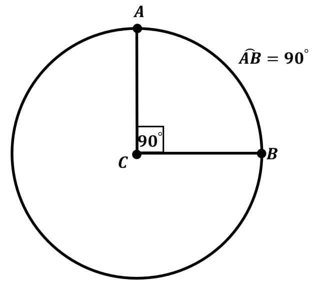
How to Prove Circles Congruent ≅:

Circles are equal if they have congruent radii, diameters, circumference, and/or area.

Parts of a Circle:



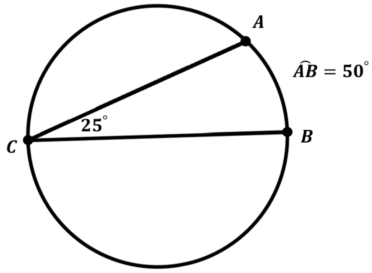
Central Angles = Measure of Arc



$\measuredangle ACB = \widehat{AB}$

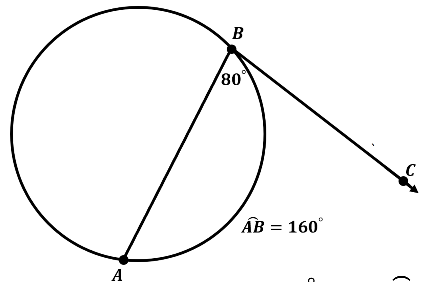
$\measuredangle ACB = 90^\circ$ and $\widehat{AB} = 90^\circ$

Inscribed Angle = $\frac{1}{2}$ Arc



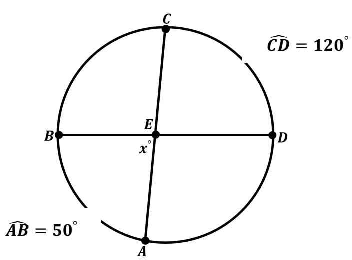
$\measuredangle ACB = 25^\circ$ and $\widehat{AB} = 50^\circ$

Tangent/Chord Angle =$\frac{1}{2}$ *Arc*



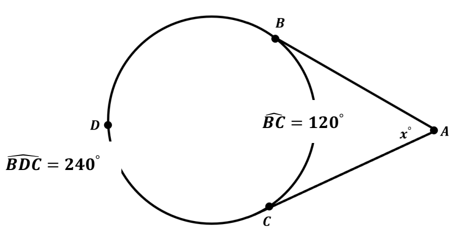
$\measuredangle ABC = 80^\circ$ and $\widehat{AB} = 160^\circ$

Angle formed by Two Intersecting Chords = $\frac{1}{2}$ *the sum of Intercepted Arcs*



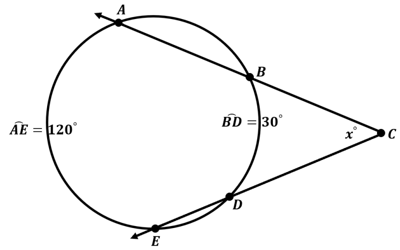
$$\measuredangle BEA = \frac{1}{2}(\ \widehat{AB} + \widehat{CD}\ )$$
$$\measuredangle BEA = \frac{1}{2}(120^\circ + 50^\circ)$$
$$\measuredangle BEA = \frac{1}{2}(170^\circ)$$
$$\measuredangle BEA = 85^\circ$$

Angle formed by Two Tangents = $\frac{1}{2}$ *the difference of Intercepted Arc*



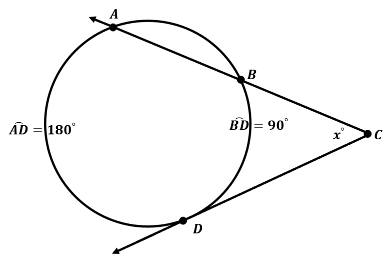
$$\measuredangle BAC = \frac{1}{2}(\ \widehat{BDC} - \widehat{BC}\ )$$
$$\measuredangle BAC = \frac{1}{2}(240^\circ - 120^\circ)$$
$$\measuredangle BAC = \frac{1}{2}(120^\circ)$$
$$\measuredangle BAC = 60^\circ$$

Angle formed by two Secants = $\frac{1}{2}$ *the difference of Intercepted Arc*



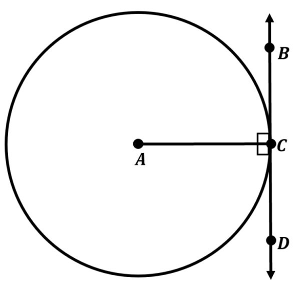
$$\measuredangle ACD = \frac{1}{2}(\widehat{AD} - \widehat{BE})$$
$$\measuredangle ACD = \frac{1}{2}(120^\circ - 30^\circ)$$
$$\measuredangle ACD = \frac{1}{2}(90^\circ)$$
$$\measuredangle ACD = 45^\circ$$

Angle formed by a Secant and Tangent = $\frac{1}{2}$ *the difference of Intercepted Arc*



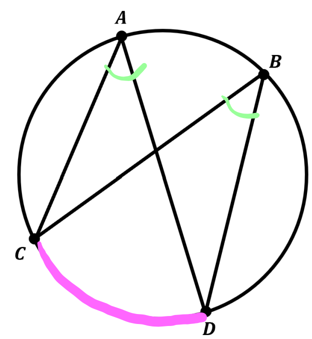
$$\measuredangle ACD = \frac{1}{2}(\widehat{AD} - \widehat{BD})$$
$$\measuredangle ACD = \frac{1}{2}(180^\circ - 70^\circ)$$
$$\measuredangle ACD = \frac{1}{2}(110^\circ)$$
$$\measuredangle ACD = 55^\circ$$

Circle Theorems:



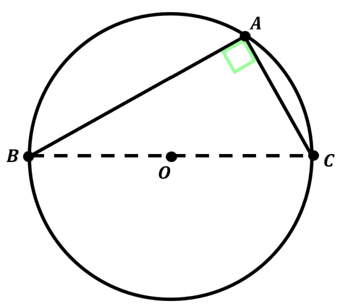
In a circle when a tangent and radius come to touch, they form a 90° angle.

$\measuredangle ACB = 90^\circ$ and $\measuredangle ACD = 90^\circ$



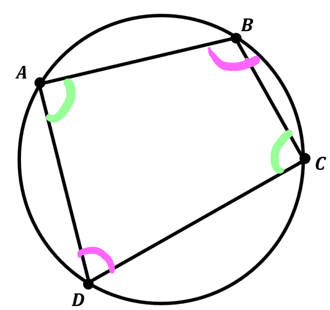
In a circle when two inscribed angles intercept the same arc, the angles are congruent.

$\measuredangle A \cong \measuredangle B$



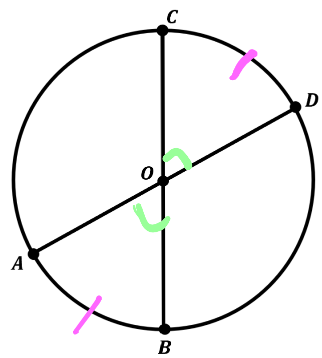
In a circle when an angle is inscribed by a semicircle, it forms a 90° angle.

$\measuredangle BAC \cong 90^\circ$



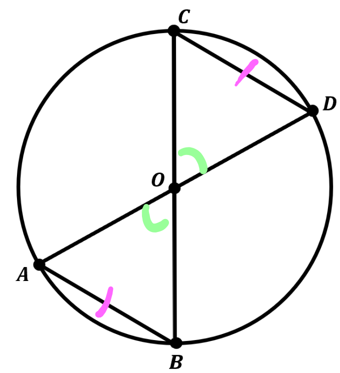
When a quadrilateral is inscribed in a circle, opposite angles are supplementary.

$\measuredangle A + \measuredangle C = 180^\circ$ and $\measuredangle B + \measuredangle D = 180^\circ$



In a circle when central angles are congruent, arcs are also congruent. (and vice versa)

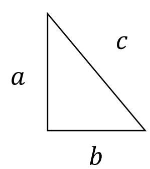
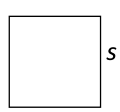
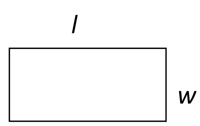
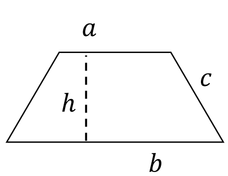
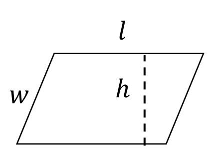
$\measuredangle COD \cong \measuredangle AOB$ Therefore, $\widehat{AB} \cong \widehat{CD}$

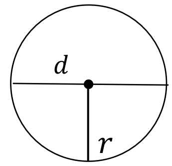
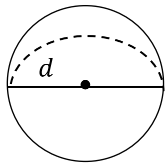
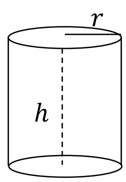
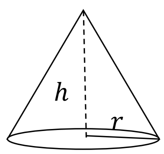
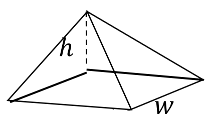
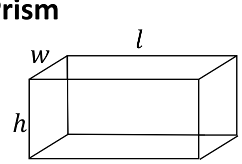
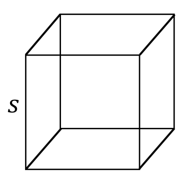


In a circle when central angles are congruent, chords are also congruent. (and vice versa)

$\measuredangle COD \cong \measuredangle AOB$ Therefore, $\widehat{AB} \cong \widehat{CD}$

Perimeter, Area and Volume:

Shape	Perimeter	Area	Volume
Triangle 	$P=a+b+c$	$A = \frac{1}{2}ab$	
Square 	$P=4s$	$A = s^2$	
Rectangle 	$P=2l+2w$	$A = l \times w$	
Trapezoid 	$P=a+b+2c$	$A = \frac{1}{2}(a+b)h$	
Parallelogram 	$P=2l+2w$	$A = l \times h$	

Circle 	$C = \pi d$	$A = \pi r^2$	
Sphere 		$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Cylinder 		$SA = 2\pi r^2 + 2\pi rh$	$V = \pi r^2 h$
Cone 			$V = \frac{1}{3}\pi r^2 h$
Pyramid 			$V = lw\frac{1}{3}h$
Rectangular Prism 		$SA = 2(lw + wh + lh)$	$V = l \times w \times h$
Cube 		$SA = 6s^2$	$V = s^3$